

# UNSOLVED PROBLEMS IN NUMBER THEORY AND PRIZES – A SHORT HISTORY

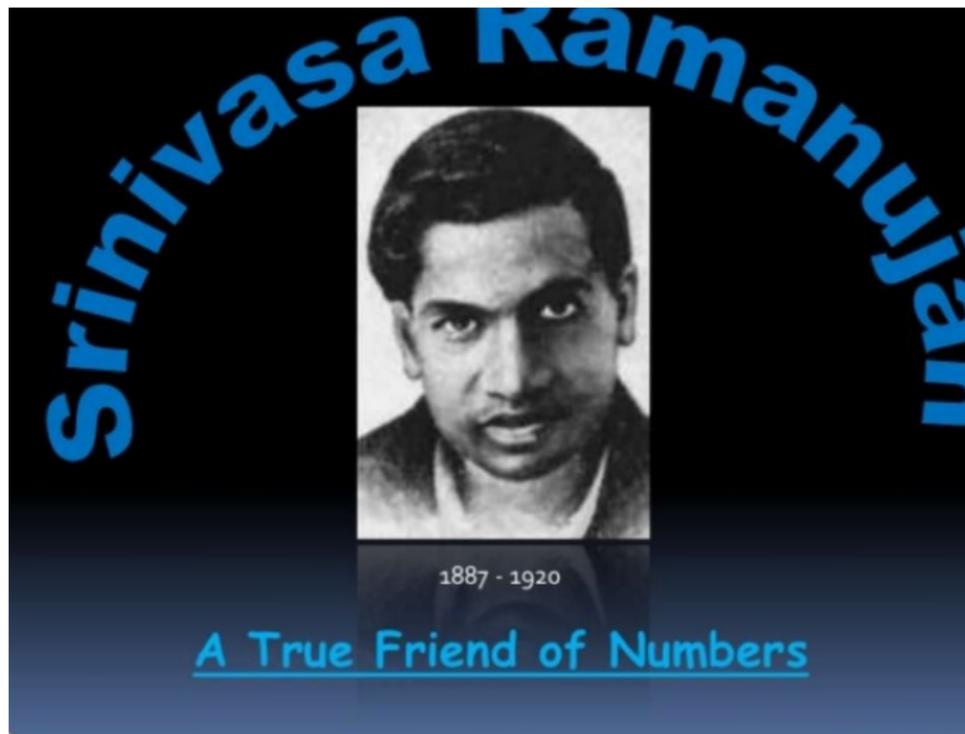
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## IN THE BRIGHT MEMORY OF



IN 1913

The English mathematician G. H. Hardy received a strange letter (ten-page) from Ramanujan.



... →



The ten-page letter contained about 120 statements of theorems in mathematics (mostly on infinite series, improper integrals, continued fractions, and number theory).

Some of the problems are ...

## DISCUSSION BETWEEN HARDY AND LITTLEWOOD

Hardy shows the letter of Ramanujan to his collaborator J. E. Littlewood.



G. H. Hardy (1877-1947)



J. E. Littlewood (1885-1977)

## CONCLUSION

After a few hours, they concluded that the results "must be true because, if they were not true, no one would have had the imagination to invent them".

## HARDY-RAMANUJAN COLLABORATION

One remarkable result of the Hardy-Ramanujan collaboration was a formula for the number  $p(n)$  of partitions of a number  $n$ . A partition of a positive integer  $n$  is just an expression for  $n$  as a sum of positive integers, regardless of order. Thus  $p(4) = 5$  because 4 can be written as  $1 + 1 + 1 + 1$ ,  $1 + 1 + 2$ ,  $2 + 2$ ,  $1 + 3$ , or 4.

## HARDY-RAMANUJAN'S ASYMPTOTIC SOLUTION (1918)

$$p(n) \approx \frac{1}{4n\sqrt{3}} e^{\pi\sqrt{2n/3}}.$$

## DEGREE AND HONOR

Cambridge granted Ramanujan a Bachelor of Science degree "by research" in 1916, and he was elected a Fellow of the Royal Society (the first Indian to be so honored) in 1918.

In 1997, **The Ramanujan Journal** (published by **Springer**) was launched to publish work "in areas of mathematics influenced by Ramanujan".

# RAMANUJAN AND FERMAT'S LAST THEOREM

## FERMAT'S LAST THEOREM (1637)



P. de Fermat (1607–1665)



A. J. Wiles (1953-)

Looks for three positive integer solutions to  $a^n + b^n = c^n$ ,  $n > 2$ .

ANDREW WILES (A BRITISH MATHEMATICIAN) PROVED IN 1995

*no three positive integers  $a$ ,  $b$ , and  $c$  can satisfy the equation  $a^n + b^n = c^n$  for any integer value of  $n > 2$ .*

### THE TAXI-CAB NUMBER OF HARDY-RAMANUJAN: 1729

The two ways to form 1729 can be matched as

$$10^3 + 9^3 = 12^3 + 1^3.$$

In other words  $(10^3 + 9^3 = 12^3)$  is a near miss solution to Fermat's Last Theorem, as it only misses by 1.

Ramanujan identified a way to generate an infinite number of near misses of the form:  $x^3 + y^3 = z^3 \pm 1$ .

### EXAMPLES (RAMANUJAN)

$$10^3 + 9^3 = 12^3 + 1^3;$$

$$6^3 + 8^3 = 9^3 - 1;$$

$$135^3 + 138^3 = 172^3 - 1;$$

$$791^3 + 812^3 = 1010^3 - 1;$$

$$11161^3 + 11468^3 = 14258^3 + 1.$$

## HONOR TO ANDREW WILES

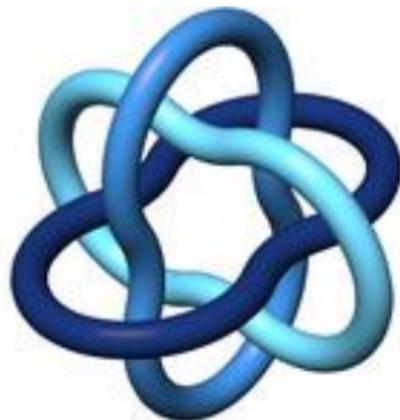
Andrew Wiles was 41, when he solved the famous Fermat's Last Theorem. Since then he honored with several (12) prestigious international prizes including Fermat Prize (1995) and Abel Prize (2016).

## IT'S A DISAPPOINTMENT!

Wiles could not get a Fields Medal! Could have awarded with Fields Medal, if he would have proved Fermat's Theorem just before one year.

## NOBEL PRIZES IN MATHEMATICS

Fields Medal and Abel Prize are the most prestigious prizes in mathematics equivalent to the Nobel Prize.



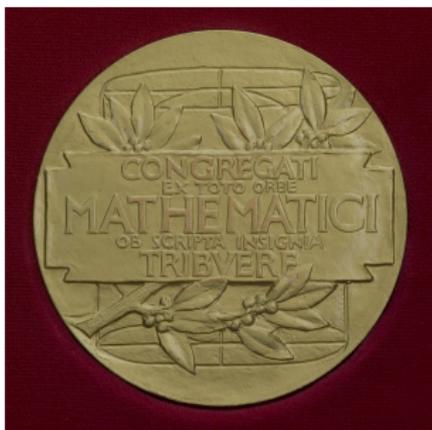
**International Mathematical Union, Berlin, Germany**



John Charles Fields  
(1863-1932), a Canadian  
mathematician



A photograph of his gravestone in  
Hamilton, Ontario, Canada



Fields Medal Back



Fields Medal Front

The Fields Medal is made of gold and shows the head of Archimedes (287-212 BC) together with a quotation attributed to him: "Transire suum pectus mundoque potiri" ("Rise above oneself and grasp the world"). The reverse side bears the inscription: "Congregati ex toto orbe mathematici ob scripta insignia tribuere" ("The mathematicians assembled here from all over the world pay tribute for outstanding work").

In contrast with the Nobel Prize, the Fields Medal is awarded only every four years (contributions are counted yearly basis). The Medal also has an age limit: a recipient's 40th birthday must not occur before 1 January of the year in which the Fields Medal is awarded. This was first awarded in 1936, and since 1950 has been awarded every four years at the International Congress of Mathematicians (ICM).

The first two medals were awarded to

- Lars Ahlfors (Finnish mathematician)
- Jesse Douglas (American mathematician)

at Oslo in 1936.

## ANDREW WILES' MOST SUCCESSFUL STORY IS

### HIS DOCTORAL STUDENT MANJUL BHARGAVA (1974-)



An Indian-American mathematician received SASTRA Ramanujan Prize in 2005 and Fields Medal in 2014.

### THE SASTRA RAMANUJAN PRIZE

Founded by Shanmugha Arts, Science, Technology & Research Academy (SASTRA) University in Kumbakonam, India, Srinivasa Ramanujan's hometown, is awarded every year to a young mathematician judged to have done outstanding work in Ramanujan's fields of interest. The age limit for the prize has been set at 32 (the age at which Ramanujan died), and the current award is \$10,000.

## THE ABEL PRIZE



ABEL  
PRISEN

- Since 2003, the Abel Prize has been awarded annually by the Norwegian Academy of Sciences, which also awards the Nobel Peace Prize.
- It is given for “outstanding scientific work in the field of mathematics”, which has meant a lifetime achievement.
- So, it is for older mathematicians.



Niels Henrik Abel (1802-1829),  
a Norwegian mathematician



Gustav Vigeland's Abel  
Monument, Oslo, Norway

### EXAMPLES OF ABEL PRIZE WINNERS

2003: Jean-Pierre Serre (French mathematician)

2007: Srinivasa S. R. Varadhan (Indian-American mathematician)

2009: Mikhail Gromov (Russian-French mathematician)

2015: John F. Nash (American mathematician)—**also received Nobel Prize in 1994**

2016: Andrew John Wiles (British mathematician)



Srinivasa Varadhan in 1947



Srinivasa Varadhan in 2007

**Srinivasa Varadhan (1940-), an Indian-American mathematician**

## ONE CAN GET FIELDS MEDAL/ABEL PRIZE IF

### HE/SHE SOLVES AT LEAST ONE OF

- The Millennium Prize Problems—introduced by—The Clay Mathematics Institute of Cambridge, Massachusetts (CMI)
- Hilbert's Eighth Problem – introduced by—David Hilbert, A German mathematician (A list of 23-unsolved problems presented on August 8, 1900 in ICM held at Paris)

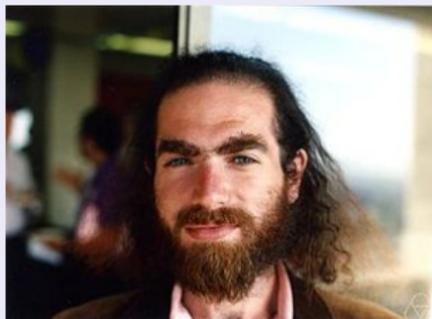
The Riemann Hypothesis is common to both, but Goldbach's Conjecture is part of Hilbert's Eighth Problem

### THE FAMOUS GOLDBACH'S CONJECTURE

Every even integer greater than 2 can be expressed as the sum of two primes.

A FACT—PROVEN BY GRIGORI PERELMAN (1966-)

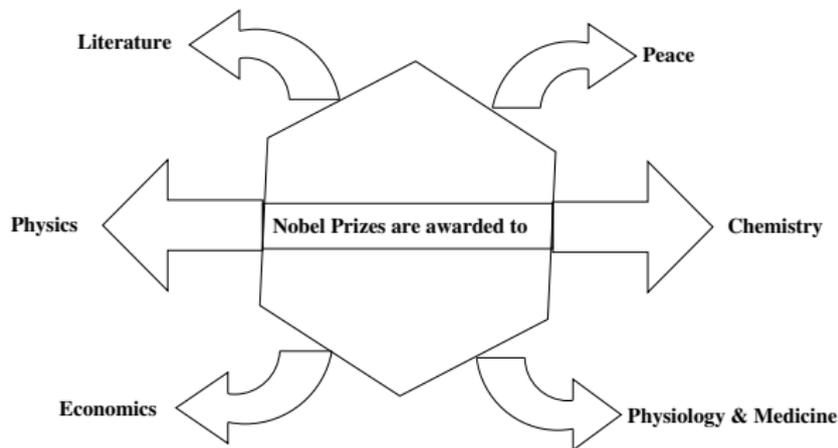
But, nobody knows why the great mathematician



**Grigori Perelman**, a Russian mathematician

declined to receive the awards EMS (European Mathematical Society) Prize (1996), Fields Medal (2006), and Millennium Prize (2010) after solving the *Poincaré Conjecture* (in 2003), one of the US \$ 1, 000, 000 problems in mathematics.

## BUT ONE CAN NOT GET NOBEL PRIZE, BECAUSE



**There is no Nobel Prize in Mathematics !**

## SOME MATHEMATICIANS DISPROVED !

- John Forbes Nash (American) – Nobel Prize in 1994 (Economics). **Nash is the subject of the 2001 Hollywood movie *A Beautiful Mind*.**
- Bertrand Russell (British) – Nobel Prize in 1950 (Literature).
- Max Born (German-British) – Nobel Prize in 1954 (Physics).



Nash  
(1928-2015)



Russell (1872-1970)



Born (1882-1970)

# WHY THERE IS NO NOBEL PRIZE IN MATHEMATICS ?



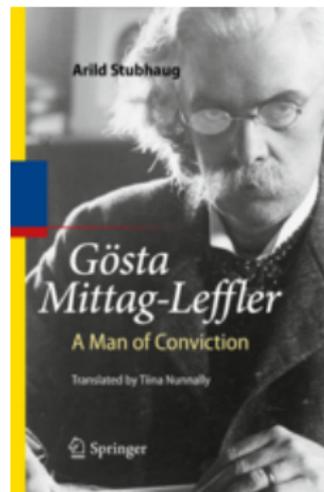
Alfred Nobel (1833-1896), a Swedish chemist. In 1863, Nobel left St. Petersburg and moved to Stockholm.



Gösta Mittag-Leffler (1846-1927), a Swedish mathematician. The first professor in mathematics at Stockholm University (1882-1911).



Sofia Kovalevskaya (1850, Russia-1891, Sweden)



**Gösta Mittag-Leffler: A Man of Conviction**  
**Arild Stubhaug (Author)**  
**Tiina Nunnally (Translator)**

WHO KNOWS? IF RAMANUJAN WERE ALIVE, PERHAPS HE WOULD HAVE WON ALL THE PRIZES: FIELDS MEDAL, ABEL PRIZE, AND MANY MORE... INCLUDING THE NOBEL PRIZE !

LET'S WATCH A DOCUMENTED MOVIE AND IMAGINE !

Thank You